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Origin of Rate Dependence in Frictional Sliding

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Experiments indicate that frictional resistance to sliding between macroscopic, clean, dry surfaces depends on the average rate V at which the surfaces are translated relative to each other. Using a new lattice automaton, we obtain results suggesting that rate-dependent macroscopic dynamics may arise from microscopic interactions between contact points which decay from a metastable state with a finite lifetime Γ. Sliding is accommodated by clusters, or avalanches, of failed lattice contact points, and corresponds to successive quenches into the metastable state by an electromechanical loading system with a finite response time Δ. Under the quasistatic assumption $\Delta \gg \Gamma$, rate dependence is a consequence of the increase in correlation length ξ_d of clusters of failed lattice points as quench rate increases. Special cases of the model are isomorphic to the self-organized criticality model for sandpiles, and to block-spring models of the type first studied by Burridge and Knopoff for earthquakes.

KEY WORDS: Friction; critical phenomena; earthquakes; self-organized criticality; scaling.

Laboratory experiments of frictional sliding between clean, dry, macroscopic surfaces translating in relative motion indicate that macroscopic friction stress depends on the average rate V at which the two surfaces slide over each other.[1-4] These experiments indicate that for small velocities V, increasing V leads to decreasing friction force F. This effect is termed "velocity weakening," and is usually ascribed to a variety of rate-dependent deformation processes operating at the microscopic scale on the sliding surfaces.[1-4] In typical experiments, V ranges from a few tenths to a few thousands of micrometers/second. Sliding can be either unstable, in which

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a portion of the relative slip occurs in discontinuous jumps, or stable, in which the relative slip is a continuous function of time. By contrast, atomic force microscope (AFM) experiments of a tungsten tip sliding on a graphite substrate indicate no evidence of rate dependence[5] over the velocity range $V = 40$–4000 Å/sec. The friction force F undergoes a transition from stable sliding to stick-slip as the normal load on the tip is increased, but in all cases F is a periodic function of the reference point displacement Vt (t is time). Spatial periodicity is equal to that of the atomic lattice. An important question is therefore to reconcile the results showing that F can be a spatially periodic function of slip on the microscopic scale, and yet be a function of V on the macroscopic scale.

In this paper, we present a model for frictional sliding that reproduces several important aspects of observed experiments, and moreover provides clear physical interpretations for all relevant macroscopic variables. Our starting point is a lattice automaton model generalized from earlier models developed in the seismological literature.[6] The model idealizes the contacting surface as a lattice of microscopic contact points, called "asperities." Sliding of the surfaces occurs when, upon successive application of a shearing force at a series of discrete times, contact points fail and slip relative to each other. An important component of our model is the existence of two time scales, which we call Γ and Δ. The former is defined as the average time for an asperity to fail upon application of sufficient shearing force. The latter is defined as the average time interval between successive application of shearing force. The interval Δ has a readily measurable value and a clear physical meaning, described below. We argue that Γ is in fact equivalent to the average lifetime of an asperity in a state of metastable equilibrium, and that V is an average quench rate. Thus we regard the sliding process as possessing similarities to nucleation and critical phenomena.

We find that our model can explain the macroscopic laboratory observations of rate dependence in the limit when $\Delta \gg \Gamma$. A significant result is the appearance of macroscopic rate dependence similar to the laboratory experiments, despite the lack of any rate dependence at the microscopic scale. Thus our model suggests that the macroscopic dynamics (the rate dependence) of the system may be a result of the interplay between the laboratory apparatus and the sliding surfaces. That the limitations of an experimental apparatus can influence the outcome of an experiment is well understood (e.g., quantum mechanics).

Typical experiments are conducted as shown schematically in Fig. 1 (top), in which a sample block of mass M is dragged over a contacting surface at velocity $V(t)$ by a servo-controlled ram-and-piston arrangement, represented by the spring with constant K. The normal force N_F on the

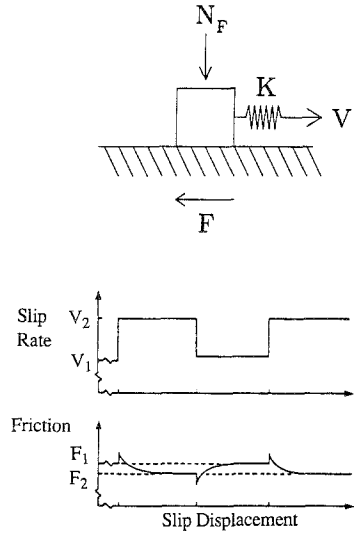

Fig. 1. Top: Block sliding over frictional surface. Middle and bottom: Changes in V produce changes in frictional resistance (after ref. 2).

block is specified, and the sliding surfaces are prepared by grinding to a known roughness. If the sliding rate V is small enough that inertial forces may be neglected during steady sliding, the ram force is equal to the friction force F resisting sliding. The relative sliding rate between the surfaces is monitored by displacement gauges spanning the contacting surfaces, the servo-controller adjusting the ram force so as to produce the specified velocity as a function of time t. The servo-controlled electromechanical driving system has a characteristic response time interval of the order of milliseconds, over which the adjustment occurs. We argue that this electromechanical response time interval plays the same role in the laboratory as does Δ in our model.

Within a certain range of sliding rate, typically $V = 0.1$–$100\ \mu$m/sec, a sudden increase $\delta V > 0$ in V typically produces: (1) a sudden, sharp peak in the observed force F required to pull the block, followed by (2) a gradual decline in F to a new value lower than before (Fig. 1). A sudden decrease by δV produces the inverse effect. Current models for describing this effect over a small range in V postulate the existence of one or more state variables $\theta_n(t)$. Models with one state variable $\theta(t)$ have been proposed[1] in which $\theta(t)$ represents the average asperity contact time. However, other work[2] indicates that at least two state variables $\theta_n(t)$ $(n = 1, 2)$ are needed to adequately model effect 2), and more state variables produce an even better data fit. Thus the contact time interpretation of $\theta(t)$ cannot be correct.

An example of a rate-dependent friction model is[2]

$$F = F_* + A_* \log(V/V_*) + \sum_n \theta_n$$
$$d\theta_n/dt = -(V/D_n)[\theta_n(t) + B_n \log(V/V_*)] \tag{1}$$

There are $3+2L$ constants, (F_*, A_*, V_*) and $(D_n, B_n; n = 1,..., L)$, which must be determined by experiment. The D_n are characteristic distances over which θ_n evolves, and are typically 1–100 μm for laboratory experiments, crudely similar in magnitude to the grit size used to grind the surfaces. It is also known that $A_* - B_n < 0$ describes sliding with instabilities, whereas $A_* - B_n > 0$ characterizes stable sliding.[2-4] Convincing physical interpretations for B_n, F_*, A_*, and V_* are lacking at present. Moreover, the physical meaning of the state variables θ_n must be identified as well.

The important questions we address in this paper are (1) to reconcile the observations of macroscopic rate dependence of frictional strength with microscopic rate independence, and (2) the physical meaning of the state variables. While the model we present below suggests answers to both these questions, it is not clear that these answers are unique.

We propose that rate-dependent macroscopic friction arises as a cooperative effect of the rate-independent *cohesion–decohesion* occurring at all of the microscopic contact points between the sliding surfaces. By cohesion–decohesion we mean the process in which two contact points (asperities) on the opposing surfaces stick together until the local force becomes large enough to break the contact, and cause the corresponding points on the two surfaces to jump laterally apart to new local equilibrium positions. This process can also be called microscopic "stick-slip" friction. Below we present a conceptually simple lattice automaton model that illustrates the cooperative nature of macroscopic friction. Our automaton replaces details of microscopic cohesion–decohesion by a simple jump rule. The jump rule provides the specification for calculating the slip undergone by pairs of microscopic contact points subject to macroscopically applied forces. While our model is simple, it nevertheless displays spatial periodicity of F on the microscopic scale, and rate dependence of F on the macroscopic scale. The latter arises as a consequence of the rate dependence of the correlation length ξ_d for clusters of failed asperities near a dynamical critical point.

We proceed by erecting a lattice dividing the bottom surface of the macroscopic block in Fig. 1 into an $M \times M$ lattice of adjoining microscopic squares. The force σ_i on the ith square is[7]

$$\sigma_i(t) = p_i(t) + \sum_j T_{ij}\phi_j(t) \tag{2}$$

where p_i is an externally applied force, and T_{ij} is an interaction between squares (Green's function). The order parameter $\phi_i(t) = s_i - Vt$ is the "slip deficit," with s_i being the total distance square i has slipped. A failure envelope σ_i^F is specified, so that slip occurs if $\sigma_i > \sigma_i^F$. Following ref. 8, we define patch i to be in an active (metastable) state having a lifetime Γ when $\sigma_i > \sigma_i^F$, and in a passive state when $\sigma_i \leqslant \sigma_i^F$. There are two critical assumptions in the model: (1) *both the lifetime Γ and the response time Δ are independent of V, and* (2) *$\Delta \gg \Gamma$, so that the surfaces slide quasistatically and are thus at equilibrium during most of the increment Δ.* The dynamics is generated by incrementing t by $\delta t = \Delta$, producing a force increment $\delta\sigma_i = -\sum_j T_{ij} V\Delta > 0$ at site i at regular intervals δt. As these sites fail by sliding, a chain reaction or avalanche of failed sites may occur by triggering of neighboring sites. For our numerical calculations, we put $\delta t = \Delta = 1$.

The final specification in the model is the jump rule[6,7] by which a site decays from the active to the passive state. As an example, suppose that $V = 0$, so that (2) becomes $\sigma_i(t) = p_i(t) + \sum_j T_{ij} s_j(t)$. Use $\sigma_i^F = \sigma^F = \text{const}$, and assume that T_{ij} is an interaction whose self-interaction term is $T_{\langle s \rangle} = -4$ and whose nearest-neighbor interactions are $T_{\langle nn \rangle} = +1$. The dynamics is generated by increasing p_i until one or more $\sigma_i \geqslant \sigma^F$. At that point, all sites in the active state are updated by the jump rule $s_i \to s_i + 1$. When s_i increases by 1, $\sigma_i \to \sigma_i - 4$, and $\sigma_{\langle nn \rangle} \to \sigma_{\langle nn \rangle} + 1$. If σ_i is identified with z_i in ref. 8, it is clear that the dynamics is identical to the self-organized criticality (SOC) model for sandpiles.[8] While the SOC model is distinguished by having a divergent correlation length $\xi_d \to \infty$ in the self-organized critical state, the models we discuss below, which have different T_{ij} and different jump rules, have finite ξ_d that depend on V.

More general models with finite ξ_d are obtained when the zeroth moment $T^{(0)} = \sum_j T_{ij} < 0$, for different jump rules. Our results (i.e., scaling exponents and statistics) are insensitive to the choice of jump rule, indicating the possibility of universality for given T_{ij}. For models with $T_{\langle s \rangle} < 0$, $T_{\langle nn \rangle} > 0$, and $T^{(0)} < 0$, there is an appealing physical interpretation, the two-dimensional massless Burridge–Knopoff model.[9] A network (Fig. 2) of massless microscopic blocks representing the lattice patches are connected together by coupling springs with constant $K_L = -T^{(0)}$. Each microblock is also connected by a loading spring $K_C = T_{\langle nn \rangle}$ to a rigid loader translating at velocity V. Motion of the microblocks is resisted by friction on the sliding surface. Each microblock represents an asperity on the underside of the macroscopic block of Fig. 1.

The jump rule used below is motivated by the observation that many physical properties of natural surfaces are fractal[10] with dimension D in the range 2.1–2.5. For example, the topography of a single surface, as well as the topographic separation between surfaces, display this property. In

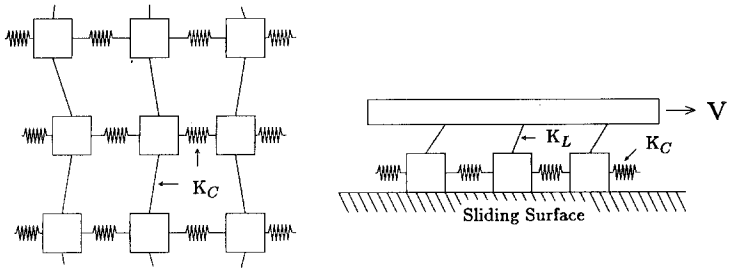

Fig. 2. Burridge–Knopoff model.

other words, there are long-range correlations in surface properties characterized by power-law spatial dependence. It is logical to assume that cohesion between surfaces is also characterized by a power law with a similar fractal dimension. It was therefore assumed that σ_i^F is a fractal, and a failure surface σ_i^F was generated[11] with $D = 2.2$, bounded by the limits $50 \leqslant \sigma_i^F \leqslant 350$. After each time increment Δ, stress at all lattice points is checked sequentially, in a series of iterations. By assumption, each iteration takes one relaxation time Γ. In each iteration, if a site is in the active state, its slip s_i is updated by an amount $\delta s_i = \{(\sigma^D - \sigma_i)/T_{\langle s \rangle}\}$, where σ^D is the dynamic frictional, or healing, strength. In our numerical calculations, $\sigma^D = 10$. Thus, each microblock undergoes simple frictional stick slip (cohesion–decohesion). We also experimented with two other jump rules. In the first, σ_i^F is a random function of position (cohesion is spatially uncorrelated). In the second, $\sigma_i^F = \text{const}$, and the slip increment is obtained by moving the microblock forward to a local minimum potential energy position plus a random overshoot. *The resulting macroscopic dynamics (statistics, dependence on K_C and K_L, and scaling exponents) were identical in all significant respects.*

To summarize our automaton, we have the following rules:

1. Begin from random initial conditions $s_i(0)$.
2. Increment time t by $\delta t = \Delta = 1$.
3. Assuming $T_{\langle s \rangle} < 0$, $T_{\langle nn \rangle} > 0$, and $T^{(0)} < 0$, calculate the local stress $\sigma_i(t)$ at each site using (2) with $p_i(t) = 0$:

$$\sigma_i(t) = \sum_j T_{ij} s_j(t) - Vt \sum_j T_{ij} \tag{3}$$

4. With t fixed, pass through the lattice to locate all active sites with

$\sigma_i \geqslant \sigma^F$, and adjust the slip of all such sites, $s_i \to s_i + \delta s_i$. By assumption, one iteration takes a time interval Γ.

5. Repeat step 4 until no active sites remain.
6. Repeat steps 2–5 until t increases to a preset limit.

Our automaton model is consistent with the results showing that F can be a spatially periodic function of slip on the microscopic scale, and yet be a function of V on the macroscopic scale. To see this, first consider the microscopic dynamics of our model, and assume $T_{\langle s \rangle} < 0$, $T_{\langle nn \rangle} > 0$, and $T^{(0)} < 0$. Using only one pair of contacts across the surfaces gives a microscopic friction force $F = \sigma_i = [T_{ii}\phi_i] = |T_{\langle s \rangle}|\,(Vt - s_i)$. As t is incremented by Δ, σ_i increases in units of $\delta\sigma_i = |T_{\langle s \rangle}|\,V\Delta$ until $\sigma_i = \sigma_i^F$. At that point, $s_i \to s_i + (\sigma_i^F - \sigma^D)/|T_{\langle s \rangle}|$, and $\sigma_i \to \sigma^D$. Following this jump to a new state, σ_i again begins to increase by $\delta\sigma_i = |T_{\langle s \rangle}|\,V\Delta$, and the process repeats. The result is that F is a periodic (sawtooth) function of reference point displacement Vt, similar to results of the AFM experiments.[5]

For macroscopic dynamics, the force is not a simple function of Vt. Here the dynamics generates avalanches or clusters of failed sites which appear on each time step (Fig. 3). The spatial average of slip rate on the clusters fluctuates closely around V. To illustrate this, we performed a calculation for a particular model in which $T_{\langle s \rangle} = -5$, $T_{\langle nn \rangle} = 1$, and $T^{(0)} = -1$. If the rate of occurrence $N(S)$ of clusters of size S is plotted against S (Fig. 4), a scaling interval is observed, $N(S) \propto S^{-\tau}$ with Fisher exponent $\tau \approx 1.6$. With increasing V and fixed $\delta t = \Delta$, the number of failed sites must increase to provide the extra slip at each time step. Both the rate of occurrence of clusters of a given size increases, as well as the critical cluster size S_c terminating the scaling region (Fig. 4). It is reasonable to

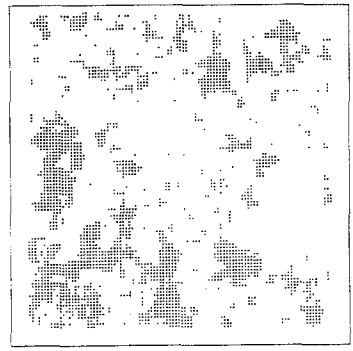

Fig. 3. Clusters of failed lattice points for 100×100 lattice on time step 20101. Each dot represents a failed lattice point.

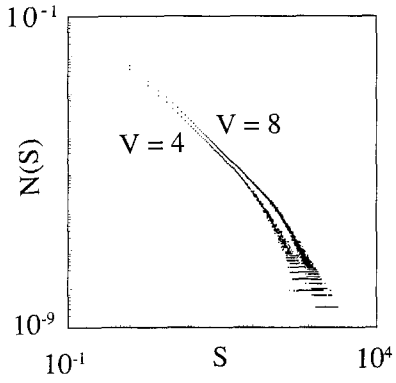

Fig. 4. Frequency of occurrence $N(S)$ per lattice site per time interval of clusters of size S, for calculation shown in Fig. 3.

expect[11] that the mean cluster size $\langle S \rangle$, and therefore also the critical cluster size S_c, is a power law function of the dynamic correlation length ξ_d, $S_c \propto \xi_d^{D_c}$, where D_c is an "average" fractal dimension, $D_c > 0$. Therefore ξ_d is an increasing function of V.

In the empirical rate-dependent models (1), the initial change δF in F following a sudden increase from V_1 to V_2 is $\delta F = A_* \log(V_2/V_1)$, a sharp increase. The narrow peak in F (Fig. 1) is therefore treated in (1) as a property of friction. However, it is clear from Newton's law that a sudden change in momentum of a real block, $M(V_2 - V_1)$, over a short time δt demands that the initial force be impulsive. Thus, the term $A_* \log(V/V_*)$ must arise at least in part from inertia.

Our calculations indicate that the long-term dependence of friction F on V occurs through the dependence on cluster frequency $N(S)$. Let $F = \sigma(t)$ denote the spatial average of $\sigma_i(t)$. Following an increase from V_1 to V_2, $\sigma(t)$ at the end of each δt (relaxed value σ_R; Fig. 5) is on average lower for V_2 than for V_1. However, $\sigma(t)$ at the beginning of each δt (unrelaxed value σ_U) is on average higher for V_2 than for V_1. If it takes q iterations through the lattice during the interval δt to reduce all lattice sites to $\sigma_i < \sigma_i^F$, the average force $\sigma_A(t)$ observed for the interval $\delta t = \Delta$ will be

$$F = \sigma_A(t) = \frac{1}{\Delta} \int_t^{t+\Delta} \sigma(t)\, dt \approx \frac{[\sigma_U(t)\, q\Gamma + \sigma_R(t)(\Delta - q\Gamma)]}{\Delta}$$

$$\approx \sigma_R(t) \tag{4}$$

if $\Delta \gg q\Gamma$. This is called "velocity weakening," since a step increase in V leads to a decrease in F. If there can exist conditions under which $q\Gamma \approx \Delta$, then $F = \sigma_A(t) \approx \sigma_U(t)$. This would be observed as "velocity strengthening."

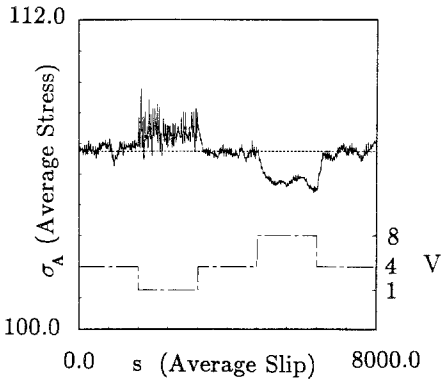

Fig. 5. Low-pass filtered change in the model stress σ_A (=friction force F) as the rate V is changed from 1 to 4 to 8.

Crossover from velocity weakening to velocity strengthening has in fact been observed in experiments.[3]

Denoting the spatial average of $\phi_i(t)$ over the lattice by $\phi(t)$, Eq. (2) indicates that a change $\delta\phi(t)$ induces a corresponding change $\delta\sigma(t)$ in average stress $\sigma(t)=F(t)$ of $\delta F(t)=\delta\sigma(t)=T^{(0)}\delta\phi(t)$. We have observed that an increase in V produces an excess number of lattice clusters for a brief duration, allowing slip on the lattice to more than catch up with the increase in V. Moreover, our model predicts that the state variables $\theta_n(t)$ in (1) are the slow modes[13] in the dynamics. Thus, $\theta_n(t)\equiv A(k_n)\,\hat{T}(k_n)\,\hat{\phi}(k_n, t)$, where $\hat{\phi}$ is the Fourier transform of ϕ_i, $\hat{T}(k_n)$ is the transform of T_{ij}, and $A(k_n)$ is a function sharply peaked about modes k_n near zero. Thus, our model also predicts that only the first few $\theta_n(t)$ will be important. Possible forms for the relaxation time spectrum include, for example, $\tau_n \propto \xi_d^z h(k_n)$, where h is a function of k_n. Internal state variables similar to $\theta_n(t)$ are often encountered in theories of material deformation[14] and may be amenable to a similar interpretation.

To summarize our results, the macroscopic rate dependence arises because increasing V corresponds to an increasing number of asperities quenched into the metastable state. Hence the correlation length ξ_d for clusters of failed lattice points increases with increasing V. Moreover, the number of clusters of all sizes increases with V. More failed points in clusters of all sizes means that average stress on the lattice is lower during most of each interval Δ. In steady sliding, the measured friction force is equal to the spatial average of lattice stress averaged over the time interval $(t, t+\Delta)$. Thus, the friction force declines as V increases. Our model leads to the prediction that the state variables $\theta_n(t)$ are proportional to the amplitudes of the slow modes of the field variable $\phi_i(t)$. Finally, we hope

that our model motivates experimentalists to search for confirmation of our ideas in laboratory data, specifically to measure the value of Γ, and to test the validity of the important assumption $\Delta \gg \Gamma$.

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